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Constitutive Modeling of Geomaterials

Advances and New Applications

Springer
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Shear Strength Criteria for Rock, Rock Joints, Rockfill, Interfaces and Rock Masses

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Summary. Although many intact rock types can be very strong, a critical confining pressure can eventually be reached in triaxial testing, such that the Mohr shear strength envelope becomes horizontal. This critical state has recently been better defined, and correct curvature, or correct deviation from linear Mohr-Coulomb has finally been found.

Standard shear testing procedures for rock joints, using multiple testing of the same sample, in case of insufficient samples, can be shown to exaggerate apparent cohesion. Even rough joints do not have any cohesion, but instead have very high friction angles at low stress, due to strong dilation.

Great similarity between the shear strength of rock joints and rockfill is demonstrated, and the interface strength between rockfill and a rock foundation is also addressed.

Rock masses, implying problems of large-scale interaction with engineering structures, may have both cohesive and frictional strength components. However, it is not correct to add these, following linear Mohr Coulomb (M-C) or non-linear Hoek-Brown (H-B) standard routines. Cohesion is broken at small strain, while friction is mobilized at larger strain and remains to the end of the shear deformation. The criterion ‘c then \(\tan \phi\)’ should replace ‘c plus \(\tan \phi\)’ for improved fit to reality. In all the above, scale effects need to be accounted for.

Keywords. Rock, rock joints, rock masses, shear strength, friction, critical state, cohesion, dilation, non-linear, scale effects.

1 Introduction

Figure 1 illustrates a series of simple empirical strength criteria that pre-date Hoek-Brown, and that are distinctly different from linear Mohr-Coulomb, due to their consistent non-linearity. Several of these categories will be addressed in this lecture and extended abstract.
Fig. 1. Simple empiricism, sometimes based on hundreds of test samples, suggested the following ways to express peak shear strength in rock mechanics and rock engineering. Note the general lack of cohesion. Derived from Barton, 1976, and Barton, 2006.

Fig. 2. Critical state line defined by $\sigma_1 = 3 \sigma_3$ was suggested by numerous high-pressure triaxial strength tests. Note the chance closeness of the unconfined strength ($\sigma_c$) circle to the confining pressure $\sigma_3$ (critical). Barton, 1976. Note that ‘J’ represents jointed rock. The magnitude of $\phi_c$ is 26.6° when $\sigma_1 = 3 \sigma_3$. 
2 Shear Strength of Intact Rock

The shear strength envelopes for intact rock, when tested over a wide range of confining stress, have marked curvature, and eventually reach a horizontal stage with no further increase in strength. This was termed the ‘critical state’ and the simple relation $\sigma_1 = 3 \sigma_3$ suggested itself, as illustrated in Figure 2. Singh et al., 2011 have now modified the Mohr-Coulomb criterion by absorbing the critical state defined in Barton, 1976, and then quantified the necessary deviation from the linear form, using a large body of experimental test data.

The Singh et al., 2011 development revealed the astonishing simplicity of the following equality: $\sigma_c \approx \sigma_3^{\text{critical}}$ for the majority of rock types: in other words the two Mohr circles referred to in Figure 2 are usually touching at their circumference. The curvature of peak shear strength envelopes is therefore now more correctly described, so that few triaxial tests are required, and need only be performed at low confining stress, in order to delineate the whole strength envelope.

![Figure 3](image.png)

**Fig. 3.** The scale-effect corrected form of the non-linear Barton 1973 strength criterion, following modification with $\phi$, by Barton and Choubey, 1977, and allowance for scale effects caused by block size. Note the strong dependence of dilation on joint properties.
3 Shear Strength of Rock Joints

Figure 3 illustrates the non-linear form of the strength criterion for rock joints. It will be noted that no cohesion intercept is intended. A linear cut-off to the origin is used at very low stress, to represent the extremely high friction angles measured at low stress. It will be noted that subscripts have been added to indicate scale-effect (reduced) values of joint roughness JRC_n and joint wall strength JCS_n. This form is known as the Barton-Bandis criterion. Its effect on strength-displacement modeling is shown in Figure 4.

4 Shear Strength of Rockfill and Interfaces

Figure 1 showed that there were similarities between the shear strength of rockfill and that of rock joints. This is because they both have ‘points in contact’, i.e. highly stressed contacting asperities or contacting opposing stones. In fact these contacting points may be close to their crushing strength, such that similar shear strength equations can apply, as suggested in Figure 5.

![Graph showing shear stress and shear displacement](image)

**Fig. 4.** Laboratory testing, especially of rough joints, may need a strong adjustment (down-scaling) for application in design, due to the block-size related scale effects on JRC and JCS. Barton, 1982
\[ \frac{\tau}{\sigma_n} = \tan [JRC \log(JCS/\sigma_n) + \varphi_r] \] applies to rock joints
\[ \frac{\tau}{\sigma_n} = \tan [R \log(S/\sigma_n) + \varphi_b] \] applies to rockfill
\[ \frac{\tau}{\sigma_n} = \tan [JRC \log(S/\sigma_n) + \varphi_r] \] might apply to interfaces

Because some dam sites in glaciated mountainous countries like Norway, Switzerland, and Austria have insufficient foundation roughness to prevent preferential shearing along the rockfill/rock foundation interface, artificial ‘trenching’ is needed. The preference for interface sliding (JRC-controlled) or failure within the rockfill (R-controlled) is illustrated in Figure 6.

Fig. 5. Peak shear strength estimates for three categories of asperity contact: rock joints, rockfill, and interfaces between the two

5 Shear Strength and Models of Rock Masses

It has been claimed – correctly – that rock masses are the single most complex of engineering materials utilized by man. The complexity may be due to variable jointing, clay-filled discontinuities, fault zones, anisotropic properties, and dramatic water inrush and rock-bursting stress problems. Nevertheless we have to make some attempt to represent this complexity in models. Two contrasting approaches (to simple cases) are shown in Figures 7 and 8.
Fig. 6. The results of interface/rockfill testing, showing R-controlled and JRC-controlled categories.

Fig. 7. Continuum and discontinuum modelling approaches to the representation of tunneling through an anisotropic rock mass. The increased richness and reality of representing the potential behaviour of jointing, even if exaggerated in 2D, is clear to see.
Fig. 8. Top: The Canadian URL mine-by break-out that developed when excavating by line-drilling, in response to the obliquely acting anisotropic stresses. This is followed by an important demonstration of unsuccessful modelling by ‘classical methods’ given by Hajiabdolmajid et al., 2000. They followed this with a more realistic degradation of cohesion and mobilization of friction in FLAC.

5.1 The Limitations of M-C, H-B and $c + \sigma_n \tan \varphi$

Attempts to model ‘break-out’ phenomena such as those illustrated in Figure 8, are not especially successful with standard Mohr-Coulomb or Hoek-Brown
failure criteria, because the actual phenomena are not following our long-standing belief in \( c + \sigma_n \tan \varphi \). The reality is degradation of cohesion at small strain and mobilization of friction (first towards peak, then towards residual) which occur at larger strain. The very important findings of Hajiabdolmajid et al., 2000 are summarised briefly by means of the six figures assembled in Figure 8. The demonstrated shortcomings of continuum modelling with \( c + \sigma_n \tan \varphi \) shear strength assumptions, should have alerted our profession for change already twelve years ago, but deep-seated beliefs or habits are traditionally hard to change.

Rock masses actually follow an even more complex progression to failure, as suggested in Barton and Pandey, 2011, who recently demonstrated the application of a similar \( c \tan \varphi \) modelling approach, but applied it in FLAC 3D, for investigating the behaviour of multiple mine-stopes in India. A further break with convention was the application of peak \( c \) and peak \( \varphi \) estimates that were derived directly from mine-logged Q-parameters, using the CC and FC parameters suggested in Barton, 2002. For this method, an estimate of UCS is required, as CC (cohesive component) and FC (frictional component) are derived from separate ‘halves’ of the formula for \( Q_c = Q \times \sigma_c / 100 \). See Table 1.

These much simpler Q-based estimates have the advantage of not requiring software for their calculation – they already exist in the Q-parameter logging data, and the effect of changed conditions such as clay-fillings, can be visualized immediately.

**Table 1.** The remarkable complexity of the algebra for estimating \( c' \) and \( \varphi \) with Hoek-Brown GSI-based formulations are contrasted with the simplicity of equations derived by ‘splitting’ the existing \( Q_c \) formula into two parts, as described in Barton, 2002.

\( (Q_c = Q \times \sigma_c/100, \text{with } \sigma_c \text{ expressed in MPa}). \)

<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FC</strong></td>
<td>( \phi' = a \sin \left[ \frac{6am_b \left(s + m_b \sigma_{3n} \right)^{a-1}}{2(1+a)(2+a) + 6am_b \left(s + m_b \sigma_{3n} \right)^{a-1}} \right] ) (from GSI)</td>
</tr>
<tr>
<td><strong>CC</strong></td>
<td>( c' = \frac{\sigma_{cl} \left(1+2a\right)s + \left(1-a\right)m_b \sigma_{3n} \left</td>
</tr>
<tr>
<td><strong>Q</strong></td>
<td>( Q = Q_c \times \frac{1}{\text{SRF}} \times \frac{\sigma_c}{100} ) (from Q)</td>
</tr>
</tbody>
</table>
Table 2. Illustration of parameters CC (MPa) and FCº for a declining sequence of rock mass qualities, with simultaneously reducing σc (MPa). Estimates of Vp (km/s) and Em (GPa) are from Barton, 2002.

<table>
<thead>
<tr>
<th>RQD</th>
<th>Jn</th>
<th>Jr</th>
<th>Ja</th>
<th>Jw</th>
<th>SRF</th>
<th>Q</th>
<th>σc</th>
<th>Qc</th>
<th>FCº</th>
<th>CC</th>
<th>Vp</th>
<th>Em</th>
</tr>
</thead>
<tbody>
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<td>2</td>
<td>1</td>
<td>1</td>
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<td>100</td>
<td>100</td>
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<td>50</td>
<td>5.5</td>
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<tr>
<td>90</td>
<td>9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
<td>100</td>
<td>45</td>
<td>10</td>
<td>4.5</td>
<td>22</td>
<td></td>
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<tr>
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<td>3.6</td>
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<td>1</td>
<td>4</td>
<td>0.66</td>
<td>2.5</td>
<td>0.1</td>
<td>33</td>
<td>0.04</td>
<td>9</td>
<td>0.3</td>
<td>2.1</td>
<td>3.5</td>
</tr>
</tbody>
</table>

An important part of the verification of the mine stope modelling reported by Barton and Pandey, 2011 was the comparison of the modelling results with the deformations actually measured.

Table 3. Empirical equations linking tunnel or cavern deformation to Q-value, with span as input (left), and the ratio of vertical stress and UCS as additional input (right). From Barton, 2002. (Note: In left equation Δ is in mm, while span remains in meters, as in left axis of Figure 9. In right equation only: Δ mm, span mm, stress and strength in consistent units, e.g. MPa).

\[
\Delta = \frac{\text{SPAN}}{Q}
\]  
(central trend of all data: approx)

\[
\Delta_V = \frac{\text{SPAN}}{100Q} \sqrt{\frac{\sigma_v}{\sigma_c}}
\]  
(more accurate estimate)

Fig. 9. The central (very approximate) data trend of tunnel deformation versus span, modified by rock mass quality Q, can be described by the simplest equation that is possible in rock engineering. See Table 3 (left side).
Fig. 10. Sample preparation, roughness profiling, tilt testing (at 1 m$^3$ scale), lowering lightly clamped sample into test frame, LVDT instrumentation, and (a rare) sheared sample. The difficulty of shearing is due to an ignored aspect of stress transformation.
CONVENTIONAL

\[ \sigma_n = \frac{1}{2} (\sigma_1 + \sigma_2) - \frac{1}{2}(\sigma_1 - \sigma_2) \cos 2\beta \]

\[ \tau = \frac{1}{2} (\sigma_1 - \sigma_2) \sin 2\beta \]

MODIFIED

\[ \sigma_n = \frac{1}{2} (\sigma_1 + \sigma_2) - \frac{1}{2}(\sigma_1 - \sigma_2) \cos 2[\beta + \delta_{n \text{ mob}}] \]

\[ \tau = \frac{1}{2} (\sigma_1 - \sigma_2) \sin 2[\beta + \delta_{n \text{ mob}}] \]

Fig. 11. Corrections for out-of-plane dilation and boundary friction, after Bakhtar and Barton, 1984
Recent reviews of pre-excavation modelling for cavern design, and actual cavern performance review for a major metro constructor in Asia, suggest that it is wise to consult these two simple equations, when deliberating over the reality (or not) of numerical models. It is the experience of the writer that distinct element UDEC-MC and UDEC-BB modellers often exaggerate the continuity of modelled jointing (because this is easier than drawing a more representative image of the less-continuous jointing, and digitising the latter). This may result in an order of magnitude error in deformation estimates.

6 A Fundamental Geotechnical Over-Sight?

This paper will be concluded with a subject that concern the transformation of stress from a principal (2D) stress state of $\sigma_1$ and $\sigma_2$ to an inclined joint, fault or failure plane, to derive the commonly required shear and normal stress components $\tau$ and $\sigma_n$. If the surface onto which stress is to be transformed does not dilate, which might be the case with a (residual-strength) fault or clay-filled discontinuity, then the assumption of co-axial or co-planar stress and strain is no doubt valid. In general this and other assumptions are not valid.
Revisiting the Paradigm of Critical State Soil Mechanics: Fabric Effects

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1 Introduction

In a recent paper Li and Dafalias [1] proposed an Anisotropic Critical State Theory (ACST) as an enhancement of the classical Critical State Theory (CST) for soils, by introducing the requirement that a fabric and loading direction related scalar-valued quantity must reach a critical state value concurrently to the classical requirement of critical state values for the stress ratio \(\eta\) and the void ratio \(e = e_c(p)\). In this process a necessary ingredient is the introduction of a measure of fabric in the form of an evolving fabric tensor, motivated by DEM simulations based on the void vectors concept presented in Li and Li [2]. The so defined fabric tensor was shown theoretically to have a critical state value norm independent of the pressure \(p\) or the specific volume \(\nu\), and dependent only on the mode of shearing via a Lode angle expression. A thermodynamic consideration of the critical state in conjunction with Gibbs’ condition of equilibrium can provide proof of uniqueness of the critical state line in the \(e-p\) space in regards to various mode of shearing. The enhanced fabric-related critical state condition can be used in a simple, if not unique, way to provide a corresponding constitutive framework for soil plasticity. The objective of this plenary presentation is to briefly outline the premises of the ACST, elaborate more on the motivation and thinking process behind the proposed theory rather than repeat several details that can be found in Li and Dafalias [1], and address several issues associated with current and future research objectives of the ACST.

2 Brief Outline of the ACST

At critical state a particulate material keeps deforming in shear at constant volume and stress. In analytical terms this is expressed by
\( \dot{p} = 0, \ \dot{s} = 0, \ \dot{\varepsilon}_v = 0 \) but \( \dot{\varepsilon} \neq 0 \) \hspace{1cm} (1)

where \( p \) is the hydrostatic pressure, \( s \) the deviatoric part of the stress tensor, \( \varepsilon_v \) the volumetric strain, \( e \) the deviatoric part of the strain tensor, and a superposed dot implies the rate. The Critical State Theory (CST) by Roscoe et al. [3] and Schofield & Wroth [4] proposes that the following conditions must hold at critical state:

\[ \eta = \eta_c = (q/p)_c = M \quad \text{and} \quad e = e_c = \dot{e}_c(p) \] \hspace{1cm} (2)

in terms of the triaxial stress variables \( p \) and \( q \), the stress ratio \( \eta = q/p \), the void ratio \( e \), and where \( M \) characterizes the intrinsic frictional coefficient between grain mineral surfaces while \( e_c = \dot{e}_c(p) \) is the critical void ratio which defines the Critical State Line (CSL) in the \( e-p \) plane. For generalization to multi-axial loading one substitutes \( \sqrt{(3/2)s_0} \) for \( q \) in the expression \( \eta = q/p \), and renders \( M \) function of the Lode angle determined by the shearing mode. In some studies under extremely small values of pressure \( p \) in microgravity [5], a dependence of \( M \) on \( p \) was observed, which might be encountered in practice for cases of liquefaction, but such variation will not be considered in this approach although it deserves a more careful examination. The classical CST makes no reference to other fabric related entities than the scalar-valued void ratio. Yet, microstructural studies without exception reveal that an intense orientational fabric formation is present at critical state [6-8], which questions the completeness of the CST.

For a particulate aggregate the fabric can be represented by appropriately defined tensors in both continuum and discrete modes. Adaptation of a discrete definition of fabric tensor \( \mathbf{G}^* \) based on the concept of void vectors by Li and Li [2] to continuum applications yields a tensor \( \mathbf{G} \) with the property that its trace measures the irrecoverable specific volume change \( v^p = 1 + e^p \), with \( e^p \) the corresponding irrecoverable void ratio change, and its deviator \( \mathbf{F} \) is exclusively a function of the orientational aspects of the aggregates arrangement with no dependence on density. Hence, one can write

\[ \mathbf{F} = \mathbf{G} - \frac{v^p}{3} \mathbf{1} \] \hspace{1cm} (3)

Henceforth, \( \mathbf{F} \) will be the fabric tensor while the change of the scalar-valued isotropic part of the fabric associated with volume change \( v^p = 1 + e^p \) will be treated as a separate internal variable. Both evolve in the course of plastic (incrementally irrecoverable) deformation. In order to distinguish between the norm \( \mathbf{F} \) and direction \( \mathbf{n}_F \) of \( \mathbf{F} \), one can write

\[ \mathbf{F} = \mathbf{F}_n \mathbf{n}_F, \ \mathbf{F} = \sqrt{\mathbf{F}^T \mathbf{F}}, \ \mathbf{n}_F: \mathbf{n}_F = 1, \ \text{tr} \mathbf{n}_F = 0 \] \hspace{1cm} (4)
In general the norm \( F \) will depend on the Lode angle \( \theta_F \) associated with \( n_F \) according to 
\[
\cos \theta_F = \sqrt{3} \text{tr} n_F^3
\]
but based on its foregoing definition will not depend on \( v' = 1 + e'' \) or the pressure \( p \).

Motivated by 2D DEM simulations such as in Li and Li [2] and Fu and Dafalias [9], it was postulated by Li and Dafalias [1] that the continuum fabric tensor tends towards a critical state value, which is characterized by a value \( F = F_c \) for the norm and a value \( n_F = n \) for the direction, \( n \) being the so-called loading direction defined by a unit-norm deviatoric tensor along, for example, the direction of the deviatoric plastic strain rate. Based on the properties of the fabric tensor defined above, it follows that \( F_c \) will depend only on \( \theta_F \). Hence, one can normalize the fabric tensor by \( F_c \) and have that at critical state \( F_c = n \) and \( F_c = 1 \) for any value of \( \theta_F \).

Motivated by an approach to account for the relative orientation of loading and fabric tensor directions presented in Li and Dafalias [10] and Dafalias et al. [11] for a fixed initial fabric tensor, Li and Dafalias [1] introduced a Fabric Anisotropy Variable (FAV) \( A = F_c \cdot n \). At critical state it follows that \( A = A_c = 1 \) since \( F_c = n \). Thus, the critical state conditions of the CST are augmented by one more related to critical state fabric in reference to current loading direction, providing the following enhanced critical state conditions of the ACST

\[
\eta = \eta_c = M, \quad e = e_c = \dot{\epsilon}_c(p) \quad \text{and} \quad A = A_c = 1 \quad (5)
\]

3 Thermodynamic Definition of Critical State and Uniqueness of CSL

The second of Eqs. (5) is the core assumption of critical state theory with or without fabric considerations, and implies the uniqueness of the Critical State Line (CSL) in \( e-p \) space. It is based on experimental evidence, which is nevertheless disputed because of the inherent difficulty to reach a critical state experimentally without localization. In Li and Dafalias [1] a thermodynamic definition of the critical state was used to prove uniqueness of CSL in conjunction with Gibbs’ condition of thermodynamic equilibrium, and the procedure is outlined below.

The critical state condition (1) states that at critical state, the shear and volumetric responses are completely decoupled, defined by a steady shear flow and a stable equilibrium volumetric state. This condition can be re-expressed within the classical thermodynamics framework by

\[
\sigma = s : \dot{e} \quad \text{and} \quad pv = C - \Psi
\]

where \( \sigma \) is the rate of the per-unit-volume dissipation; \( v \) is the specific volume; \( \Psi \) is the specific (per-unit-mass) Helmholtz free energy; and \( C \) is an energy datum independent of \( v \) (a Gibbs function). Eq. (6)_1 asserts that at critical states the shear work is
solely and completely dissipated, representing a rigid perfectly plastic condition materialized only when Eq. (2)_1 is satisfied; and Eq. (6)_2 describes an exact equilibrium condition, in which all thermodynamic properties are stationary. Eqs. (6) provide a thermodynamic definition of the critical state.

Gibbs asserts that the entropy of a system in equilibrium is the maximum among all the neighboring states with the same internal energy; or alternatively, the internal energy of an equilibrium system is the minimum among all the neighboring states having the same entropy [12]. Application of Gibbs’ equilibrium condition to Eq. (6)_2 isothermally suggests that the specific Helmholtz free energy $\Psi = u - T \vartheta$ is a minimum at critical state, where $u$, $T$ and $\vartheta$ are the specific internal energy, the absolute temperature and specific entropy, correspondingly. The specific Helmholtz free energy is in general a function of the specific volume $v$ as well as other deformation and internal variables, i.e., $\Psi = \Psi(v, \mathbb{Z})$, where $\mathbb{Z}$ represents all relevant variables other than $v$ and $p$. The fabric tensor $\mathbf{F}$ is included in $\mathbb{Z}$ since it is not dependent on $v$ or $p$.

It follows from Eq. (6)_2 and Gibbs’ equilibrium condition that for a given $p$, corresponding to a minimum $\Psi_c$ (hereafter the subscript ‘c’ stands for ‘at critical state’), $v_c$ is a maximum with respect to any neighboring states defined by variation of $\mathbb{Z}_c$ but still complying with the critical state condition Eqs. (6). Thus, solving for $p_c$ at critical state one can re-write Eq.(6)_2 as $p_c = \hat{p}_c(v_c, \mathbb{Z}_c)$ where for simplicity the datum $C = 0$. Because those neighboring states are also critical states as defined by Eqs. (6), they also demand a maximum $v_c$. Hence, the only possibility for all those neighboring critical states to satisfy the Gibbs condition is that they have the same value of $v_c$, which is the limiting case of the Gibbs condition. Since such “neighboring” critical states have their own neighboring states complying also with Eqs. (6), these “new” neighboring states are also critical states, thus, according to the limiting Gibbs condition are characterized by the same unique value of $v_c$. Clearly, such “expansion” of neighboring states can be repeated, resulting in a unique pair of $p_c$ and $v_c$ over the entire continuous domain of $\mathbb{Z}_c$. Since the fabric tensor $\mathbf{F}_c$ is one of the variables in $\mathbb{Z}_c$, it follows that the presence of fabric anisotropy at critical state has no impact on the critical state line, and accordingly the forgoing equation among $p_c$, $v_c$ and $\mathbb{Z}_c$ can be reduced to $p_c = \hat{p}_c(v_c)$. Expressing $v_c$ in terms of the void ratio $e_c$ and solving for the latter one obtains Eq.(5)_2 with $p_c = p$ as the equation of the unique CSL. Thus, at critical state the free energy expression becomes the equation of the CSL under Gibbs’ equilibrium condition.

It is important to emphasize the significance of having the fabric tensor and its norm independent of specific volume or equivalently of $p$, as defined in Li and Dafalias [1], and dependent only on the Lode angle $\theta_F$ with its subsequent normalization, in order to prove by use of Gibbs’ equilibrium condition the
uniqueness of the CSL. Any coupling of the dependence of the critical state value of the norm of the fabric tensor on specific volume and Lode angle would preclude the exact application of the proof of uniqueness of CSL. How to address such eventuality of coupled dependence will be presented in upcoming publication.

4 Evolution of Fabric Tensor

The evolution equation of $F$ will be developed within the theory of rate independent plasticity where $F$ plays the role of an evolving internal variable. Such evolution will be expressed by a corotational rate in reference to a specific constitutive spin $\omega$ in order to satisfy objectivity under large deformations and rotations, thus, one can write

$$\dot{F} = \dot{F} - \omega F + F \omega = \langle \lambda \rangle \bar{F}$$

(7)

where a superposed ‘$\dot{}$’ signifies the corotational rate in association with $\omega$, the scalar-valued $\lambda$ within the Macauley brackets is the plastic multiplier which is a function of stress or strain rate, and $\bar{F}$ is a tensor-valued isotropic function of the stress and internal variables due to objectivity.

General guidelines for the specification of the constitutive spin $\omega$ are based on the theory of Plastic Spin [13]. For small elastic deformations the theory maintains the plausible proposition that $\omega$ does not have to be equal to the continuum material spin $W$, the anti-symmetric part of the velocity gradient, hence, a plastic spin $W^p$ can be defined by $W = \omega + W^p$. The plastic spin $W^p$ is non-zero only when plastic loading occurs, i.e., when $\lambda > 0$, hence, one can write $W^p = \langle \lambda \rangle \Omega^p$ along the format of Eq. (7). Objectivity requirements render $\Omega^p$ an isotropic function of the stress and internal variables. Thus, instead of specifying the constitutive spin $\omega$ that is not objective, one can equivalently specify the $\Omega^p$ and then obtain $\omega = W - W^p = W - \langle \lambda \rangle \Omega^p$. Substitution of the last expression of $\omega$ in terms of $W$ and $\Omega^p$ in Eq. (7) yields after some simple algebra

$$\bar{F} = \dot{F} - WF + FW = \langle \lambda \rangle \left( \bar{F} + F \Omega^p - \Omega^p F \right)$$

(8)

where a superposed ‘$\dot{}$’ denotes the corotational rate with respect to $W$, otherwise known as the Jaumann rate. Therefore, one needs to specify the form of functions $\bar{F}$ and $\Omega^p$.

Recall that $F = Fn_F$ evolves towards $n$, or equivalently $F$ evolves towards 1 and $n_F$ towards $n$. The foregoing can be incorporated into Eqs. (7) and (8) by setting $\bar{F} = c(n - rF)$, which, with $r=1$ at critical state, ensures the evolution of $F$ towards $n$, where $c$ and $r$ are scalar-valued constitutive parameters (isotropic
functions of the stress and internal variables in general) controlling the pace of evolution and the peak of $F$, respectively. With $r < 1$ at a pre-critical state, $F$ may reach a peak greater than 1 before it falls to its critical state value of 1 where $r = 1$, as observed in DEM simulations for dense specimens.

For the determination of $\Omega^p$, observe first that the principal directions of $F$ will tend to rotate towards alignment with those of $n$, if such alignment does not exist to begin with. In addition recall that the $\Omega^p$ must be an isotropic function of the stress and internal variables, which in the present general development appear in the form of the loading direction $n$ and the fabric tensor $F$. Based on the representation theorems [14], the simplest form of such isotropic anti-symmetric tensor-valued function of two symmetric tensors is given by $\Omega^p = \chi (nF - Fn)$ with $\chi$ the scalar-valued plastic spin constitutive parameter (again an isotropic function of the stress and internal variables in general). Observe that when $n$ and $F$ are coaxial, i.e., they have same eigenvectors, they commute, i.e., $nF = Fn$ and, therefore, $\Omega^p = 0$. It is the non-coaxiality of $n$ and $F$ that can cause a non-zero plastic spin and by consequence a non-zero constitutive spin given by $\omega = -W^p = -\langle \lambda \rangle \Omega^p$ when $W = 0$, hence, achieving the tendency for alignment between $F$ and $n$. Substitution of $\bar{F} = c(n - rF)$ and $\Omega^p = \chi (nF - Fn)$ in Eq. (8) yields

$$\bar{F} = \hat{F}n + F\hat{n}_F = \hat{F} - WF + FW$$

$$= \langle \lambda \rangle \left[ c(n - rF) - \chi(nF^2 + F^2n - 2FnF) \right]$$

(9)

One needs the values of $c$, $r$ and $\chi$ for the implementation of Eq. (9).

5 Effect on Constitutive Modeling

The introduction of the fabric tensor $F$ and FAV $A$, while enhancing the classical CST by one additional condition as shown in Eqs. (5), it does not answer the question as to how exactly this will benefit a corresponding constitutive framework within the ACST. Of course one can treat the $F$ as an internal variable and $A$ as a joint invariant in the expressions for the free energy, the yield surface, plastic potential and dilatancy, where in combination with other internal variables and invariants can be used to describe the effects of evolving fabric anisotropy. Reaching critical state will not just be considered in the $e-p-q$ space of CST, but in the enhanced $e-p-q-A$ space of the ACST. These, however, remain general statements without practical and generic value. The issue is to address the effect of fabric in such a way as to influence directly the constitutive features associated with critical state in a generic way, i.e., in a way that can be applied to various families of constitutive models within the ACST.

A previous approach to address the fabric effect was based on the findings of several prior works by the authors, namely in Li and Dafalias [10], Dafalias et al.