Articulating Medieval Logic

TERENCE PARSONS
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Terence Parsons
To Calvin Normore
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Preface

I have benefitted greatly from discussion with the participants in several meetings of the Moody Conference over the years, as well as occasional collaborative seminars at UCLA, from participants in the 2001 Copenhagen meeting on Buridan's philosophy, in the 2003 Midwest Conference on Medieval Philosophy in Omaha, and at a pair of talks in 2006 at the University of Barcelona, from the 2007 International Conference on the Square of Opposition in Montreux, from the 17th meeting of the European Symposium for Medieval Logic and Metaphysics in Leiden in 2008, and the 19th meeting in Geneva in 2012, from talks at McGill University in 2008 and again in 2010, from a talk at the University of Nevada at Las Vegas in 2009, and another at the Society for Exact Philosophy in 2010.

I have particularly benefitted in various ways from interactions with E.J. Ashworth, Steve Barney, David Blank, John Carriero, Sten Ebbesen, Elizabeth Karger, Peter King, Gyula Klima, Henrik Lagerlund, Marko Malink, Chris Martin, John Martin, Gary Matthews, Ana Maria Mora, Catarina Dutilh Novaes, Claude Panaccio, Alex Radulescu, Stephen Read, Paul Vincent Spade, Joke Spruyt, Paul Thom, Sara Uckelman, Jack Zupko.

This book has been years in preparation. Throughout this time I have benefitted from occasional sabbaticals granted by the University of California, and I have been nurtured by my family and by my UCLA colleagues. I have been aided by discussions with my various colleagues. I owe special debts of gratitude to Brian Copenhaver and to David Kaplan, and especially to Calvin Normore for his support and insight over the years. And, of course, to my wife Anette, who claims not to understand the work I do, but in some ways understands it most of all.
Modern logic can be seen as a group of theories and practices clustered around a well-studied and well-understood paradigm theory, namely, first-order predicate logic with relation symbols and identity. This central theory is a formal logic, and formal techniques can be used to validate a vast number of arguments using only a small number of basic principles.

The main theme of this book is that medieval logic can also be seen as a group of theories and practices clustered around a core theory which is a paradigm of logic; this theory consists of a number of widely known principles, all of which can be derived from a very simple core of rules and axioms. Unlike today, however, this was not widely known, and there were only a few attempts to carry out the project of deriving most principles from a basic few. (Buridan’s TC is a prominent exception.) It has taken me some time to arrive at this view. Medieval writings by logicians can seem to consist of a variety of unsystematic and disparate remarks, and it is not at all obvious whether or how they fit together. That is what this book is about.

There are two striking differences between medieval logic and modern logic. One is the assumptions that are made concerning existential import; in medieval logic ‘every A is B’ entails that something is A. This needs to be taken seriously, and the details need to be worked out, but from a point of view of general logical principle, this difference is not a great one. The other difference is that medieval logic is formulated entirely within a natural language, Latin. This is a major constraint that needs to be respected and dealt with. It shapes the development of the theory from start to finish. To be sure, medieval Latin is a somewhat artificial natural language. In medieval times it was no longer anybody’s native tongue; everyone learned it by schooling, as a second language. However, what is important for my purposes is that it has the grammatical structure of a natural language; in particular, it obeys the "theta-criterion", as discussed in section 4.1.

Medieval logic begins from the logical theory developed by Aristotle. It is well known that Aristotle formulated a system of logic involving conversions (Some A is a B; therefore some B is an A) and syllogisms. It is also fairly well known that he assumed certain "first figure" syllogisms as axiomatic; these are argument forms such as:

\[
\text{Every } B \text{ is a } C \\
\text{Every } A \text{ is a } B \\
\therefore \text{ Every } A \text{ is a } C
\]
and he proved all of the other forms from these and the conversion principles. What is much less well known is that he did not just assume the conversion principles; he proved them. I see the techniques that he used to prove those principles as much more important and interesting than the developed system of logic for which he is known. One of these techniques is well known today: indirect derivation, or, as he called it, reductio: to prove a proposition, assume its contradictory and then derive something absurd. Two other principles are often lumped together under the Greek term 'ekthesis'. One of these is that if you are given an existential proposition, you can "choose one of the things that makes it true." In modern logic, this is existential instantiation; given $\exists x Fx$, pick some name 'n' that is not used elsewhere in the derivation and write 'Fn', and then reason from this rather than from $\exists x Fx$. In Aristotle's notation you are given something like 'some F is a G' and you pick a name 'n' not used elsewhere, and write both 'n is an F' and 'n is a G.' (In modern logic one would write the conjunction of these formulas, but Aristotle didn't bother with conjunctions.) The third technique is an analogue of our modern existential generalization: given 'Fn' one can infer $\exists x Fx$. In Aristotle's notation, given both 'n is an F' and 'n is such and such' one infers 'some F is such and such'. Using these three techniques Aristotle proved the conversion principles, and he also made occasional use of those principles in reducing some syllogisms to others. This was a major advance. What was not known then, or throughout the Middle Ages, is that using these three techniques, one may also prove all of the first figure syllogisms,1 so that those three principles provide a foundation for all of Aristotle's well-known system of logic. This is laid out in Chapters 1 and 2 of this book.

Medieval logicians inherited Aristotle's work together with propositional logic as developed principally by Stoic logicians. Chapter 3 addresses the evolution of this theory in the early 13th century. Much of the additional advances at this time were driven rather straightforwardly by expansions of the logical notation. For example, Aristotle did not allow quantified predicate terms; he argued that one should not write 'Every man is every animal' because it is not true. But 'No man is every animal' is true, and it has a quantified predicate, and logicians began using such forms freely. They also constructed sentences with negations sprinkled throughout. Once you can write things like 'Every A isn't a B' it doesn't take much to notice that this is equivalent to 'No A is a B', and logicians formulated principles for interchanging quantifier signs and negations, holding, for example, that 'No A . . .' is equivalent to 'Not some A . . .', and that 'Some A . . .' is equivalent to 'Not every A not . . .', and so on. And once singular terms are allowed to occur anywhere that a quantified common term can, it is clear that singular terms permute with negations, and with other terms, so that 'Socrates no stone is' is equivalent to 'No stone Socrates is'. And once instances of the transitivity of identity became formulable it too was recognized as a valid principle. All of this results in a rich system of logic in which a few fundamental principles permit the derivation of the rest.

1 Thom 1976.
The richness of medieval logic is especially interesting because the entire enterprise is formulated entirely within natural language—at least, within a somewhat regimented version of natural language. So this is a version of logic in which there is no logical form except for grammatical form. Logicians made this work in part by stipulating how Latin is to be understood, holding e.g. that surface order of words determines their semantic scope, so that a sentence having Latin words in this order: ‘A woman owns each cat’ is understood to have exactly one reading, meaning that there is a woman such that she owns every cat. To articulate the other reading that is possible in natural English you would have to use a Latin sentence with the word order: ‘Each cat a woman owns’, which is completely grammatical in Latin, and is stipulated to mean, unambiguously, that every cat is such that some woman owns it. This stipulation takes advantage of the relatively free word order of Latin to express quantifier scope. (It is distinctive of medieval logicians that they spend substantial time on matters of scope.) To make clear the logical theory that was developed it is essential to know the exact grammatical forms of the propositions that are employed. Chapter 4 develops a system for encoding and clarifying the grammatical structures of propositions, and there are additional expansions and applications in Chapters 5 and 6.

These expansions of the notation permit the validation of rather complex arguments, such as:

\[
\text{Some farmer's every donkey sees every horse which a merchant owns.}
\]
\[
\therefore \text{Not every horse no donkey sees.}
\]

However, the resulting system of logic, because of grammatical constraints, is still limited in its expressive resources. To become adequate one also needs anaphoric pronouns, as in ‘Some woman owns a donkey which she feeds.’ This is the task of Chapter 8. If I am right, we are confronted in the texts by two ideas about how anaphoric pronouns work. One of these—the method of singulation—is invoked as an analysis of reflexives. This is roughly the idea that anaphoric pronouns are unaffected by inferences involving their antecedents. For example, given that Socrates is a man and every man loves his mother, we infer that Socrates loves his mother, where the ‘his’ remains unchanged while its antecedent changes from ‘every man’ to ‘Socrates.’ This method works well for reflexives and for a host of other pronouns as well. There is a second method that is much discussed, and that works well in a fairly broad range of cases, but gives clearly wrong results in quite a few. This is roughly the idea that an anaphoric pronoun is an independent term; it stands for the same things as its antecedent, and has the same kind of quantificational status. As Reinhard Hülsen

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2 This needs certain qualification. For example, in Latin all of the following sentences have the same grammatical form: ‘Man is a noun,’ ‘Man is a species,’ ‘A man is a philosopher.’ In the first, the term ‘man’ is said to have “material supposition”; it is taken to stand for itself. In the second, the term is said to have “simple supposition”; it is taken to refer to a form (if there are such things; otherwise it refers to a concept or a word). In the third, the term ‘man’ is said to have “personal supposition”; it is taken to stand for individual men. All of the logic investigated in this book pertains to this last interpretation.
notes, if the first method, the one developed for reflexives, is used in place of the second method, a much better theory results. And at least some later medieval logicians did just that. With the first method, inference principles previously given yield a system of logic that is similar to the predicate calculus in richness and power. At least, this is what I argue in Chapter 9.

In addition to the topics already mentioned, we look in Chapter 7 at what is most distinctive of late medieval logic, and most well known: the useful theory of modes of personal supposition. We also examine the special terms that were used to accommodate sentences containing three or more main quantified phrases. A great deal is now known about these matters, but it is clear that this project will leave far more questions open than it answers.

Chapter 10 and the Appendix touch on further developments of the logical theory.

In focusing on principles of inference that were widely acknowledged I may fail to convey an accurate impression of the diversity of the various writings in medieval logic, and of the various ways in which authors disagreed with one another. These differences and disagreements are widely discussed in the secondary literature. Much of this is fascinating, but it is not my goal to cover it all here. Instead I focus on a few principles that are widely acknowledged and that are rarely debated. It has been suggested that I am trying to impersonate a 15th-century logician who happens to have the skills and habits of a 21st-century graduate student. I discuss known medieval views with some care, and show how far one can go without introducing any logical principles beyond the medieval ones.

I have tried to give enough quotes and citations to ground my assertions in the historical record. However, it is a skewed record. First of all, my work is based entirely on western European texts that have been edited and published, and there are a vast number of yet unedited manuscripts. Further, most of the work I report on here is available in English translation, so untranslated works are not emphasized. Fortunately, the number of first-rate English translations has now reached the point where someone can learn much about medieval logic directly without being able to read Latin. Almost all of the quotations that I give are in English. When quoting from a published translation I use the translator’s own words; otherwise I am responsible for them. Most citations to medieval works are given in terms of an abbreviation of the title of the work (e.g. ‘SD’ for ‘Summulae de Dialectica’) followed by a series of numbers, such as ‘2.3.7’. Page numbers, when given, refer to the English translation, or to the page numbers of the Latin edition if they are included in the English translation, or if there is no published translation. The texts referred to mostly date from 1200 to 1425, after Abelard and up to and including Paul of Venice, with a few later texts also discussed.

4 The meaning of the numbers will vary, depending on how the text in question is demarcated; e.g. it might mean the seventh section of the third chapter of the second book, or the seventh subsection of the third section of the second chapter. Such numbering is usually common to both the Latin edition and to the English translation.
In addition to several anonymous writings, the main logicians referred to are Peter of Spain, William Sherwood, Lambert of Lagny, Roger Bacon, Walter Burley, William Ockham, John Buridan, Albert of Saxony, Marsilius of Inghen, John Wyclif, and Paul of Venice. This group includes both metaphysical realists and nominalists. Although realists and nominalists provide semantic accounts that differ in important details, the logical principles that they endorse are pretty much the same, and so metaphysical differences are mostly not relevant. Likewise for disputes between Thomists and non-Thomists.  

It should be apparent that this book does not contain any new historical discoveries; rather it relies on discoveries by others. Over the years I have benefitted enormously from the secondary literature in learning about medieval logic. However, much of the discussion there is not directly relevant to the issues taken up here, and it would be distracting to include it—and it would lengthen this book considerably. I want to hereby acknowledge my indebtedness to the many authors who have published on this topic, and from whom I have learned.

This book is not meant as a general introduction to medieval logic. Further, there are several areas of great logical interest that are not discussed here at all: these include medieval discussion of Aristotle's topics and fallacies, insolubles (semantic and other paradoxes), obligations (rules for debates), syncategoremata (special logical principles of individual words), sophisms (logical and grammatical puzzles), exponibles (analyses of special words, such as 'only'), future contingents, consequences (meta-principles of propositional logic), and systems of modal logic (though modal sentences are discussed to some extent). These are all widely treated elsewhere.

5 For an illustration of some of the contrast between realists and nominalists see Klima 2011.
6 Aquinas did not discuss general principles of logic much. A full set of views are laid out by his follower John of Saint Thomas (Jean Poinsot) in a very competent work from the early 1600s; the logical doctrines laid out there mesh nicely with the ones discussed here. I do not cite passages from this work because it was written considerably later than the period I am discussing.
7 For a good general introduction see Kretzmann, Kenny, and Pinborg 1982; Marenbon 1987; also Lagerlund 2012, Broadie 2002, Spade 1996. Boehner 2012 is still a good overview.
8 See Stump 1982.
9 See the Introduction to Copenhaver, forthcoming, as well as Peter of Spain’s own discussions; also the Introduction to Klima 2001, as well as Buridan’s own discussions.
10 For a very brief overview see Spade 1982; also Spade 1988b, Simmons 2008, Yrjönsurri 2008.
11 For a brief overview see Stump 1982 and Spade 1982b; also Dutilh Novaes 2007.
12 For a brief overview see Kretzmann 1982.
13 For a brief overview see Kretzmann 1982; also Read 1993.
14 For a brief overview see Kretzmann 1982.
15 For a brief overview see Normore 1982.
17 For a brief overview see Knuuttila 1982 and 2008; also Thom 2003.
An Overview of Aristotelian Logic as seen by Medieval Logicians

Medieval Logic is built on a foundation of logical terminology, principles, and methodology contained in the traditional liberal arts, in particular in that part of the Trivium called Logic or Dialectic. This material is mostly from the writings of Aristotle and his commentators, plus the Stoics and others, much of it as interpreted by Boethius. This chapter and the next are devoted to these fundamental parts of logic that medieval logicians accepted as the basis of their work.

I begin with an account of the forms of propositions that constitute the subject matter of Aristotle’s symbolic logic, as understood by medieval logicians. In keeping with medieval terminology, I use the term ‘proposition’ to refer to what we today would call a meaningful sentence. It stands for a sentence with meaning, not for a sentential form, and not for an abstract meaning expressed by a sentence which is named by a that-clause. So ‘Snow is white’ and ‘Schnee ist weiss’ are different propositions.

1.1 Categorical propositions

I begin with an oversimplified account of what Aristotle had to say, mostly in the first seven sections of his short work that is today called On Interpretation, and in the first seven sections of his longer work called Prior Analytics. This is definitely not the only way to interpret Aristotle’s works, but it’s a common and straightforward one, and it fits the usual medieval explanations. I also use some standard medieval terminology which Aristotle didn’t use.

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1 The parts of Aristotle’s symbolic logic that were well known around the year 1000 were his On Interpretation, Categories, and material (often second-hand) from the first several sections of his Prior Analytics. This subject matter was later called the “Old Logic,” to distinguish it from the “New Logic” which was based on several additional writings by Aristotle that became available later.

2 Opinions differed on whether there are also mind-independent entities corresponding to propositions. For many medievals, propositions are tokens, not types, and this is important in certain cases, such as addressing semantic paradoxes, where two tokens of the same type might differ in truth value. But for the most part little would be changed in the theory if propositions were types.
In any system of logic it is essential to be clear about the vocabulary you are using, and about what propositions can be made using it. Aristotle was clear about this. The basic propositions are *categorical* propositions—meaning something like “predicational” propositions. Every such proposition consists of a subject term (perhaps modified) and a copula (perhaps with a negation) and a predicate term. The copula is ‘is.’ Every predicate term is a common noun, such as ‘donkey’ or ‘animal’ or ‘substance’ or an adjective such as ‘just.’ Every subject term is a proper noun or a common noun. There are eight forms of proposition; four affirmative and four negative. First, there are affirmative and negative “universal” propositions:

<table>
<thead>
<tr>
<th>Affirmative</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Every A is a B</td>
<td>No A is a B</td>
</tr>
</tbody>
</table>

where ‘A’ and ‘B’ are common nouns or adjectives. Examples are ‘Every donkey is an animal’ and ‘No donkey is an animal.’ In these examples I have used the indefinite article ‘a’ in order to form grammatical English propositions. There is no indefinite article in Greek, and so the indefinite article appears here as an artifact of English grammar.

There are “particular” propositions:

<table>
<thead>
<tr>
<th>Affirmative</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Some A is a B</td>
<td>Some A isn’t a B</td>
</tr>
</tbody>
</table>

Examples are: ‘Some donkey is an animal’ and ‘Some donkey isn’t an animal.’

There are “indefinite” propositions:

<table>
<thead>
<tr>
<th>Affirmative</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>An A is a B</td>
<td>An A isn’t a B</td>
</tr>
</tbody>
</table>

Examples are ‘A donkey is an animal’ and ‘A donkey isn’t an animal.’ Again, the English examples contain indefinite articles, where Greek and Latin have nothing at all. Finally, there are “singular” propositions:

<table>
<thead>
<tr>
<th>Affirmative</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>n is a B</td>
<td>n isn’t a B</td>
</tr>
</tbody>
</table>

where ‘n’ is a singular term: specifically, ‘n’ is a proper name, such as ‘Socrates,’ or a demonstrative term such as ‘this’ or ‘this animal.’ Examples are ‘Socrates is an animal’ and ‘Socrates isn’t an animal.’

---

3 It may be more true to Aristotle to say that there are two (or more) copulas, an affirmative one, ‘is,’ and a negative one ‘isn’t.’ This will not be important for the applications discussed in this chapter.

4 In the Latin texts that we will be dealing with the negation naturally precedes the verb, so the word order is ‘Some A not is B.’ Generally I use a grammatical English form. I use the contracted form ‘isn’t’ instead of ‘is not’ in order to de-emphasize the English word order, which differs from the Latin.
Table 1.1 shows the logical forms of categorical propositions:

<table>
<thead>
<tr>
<th></th>
<th>Affirmative</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Universal</td>
<td>Every A is a B</td>
<td>No A is a B</td>
</tr>
<tr>
<td>Particular</td>
<td>Some A is a B</td>
<td>Some A isn’t a B</td>
</tr>
<tr>
<td>Indefinite</td>
<td>An A is a B</td>
<td>An A isn’t a B</td>
</tr>
<tr>
<td>Singular</td>
<td>n is a B</td>
<td>n isn’t a B</td>
</tr>
</tbody>
</table>

Eventually people began describing the status of being affirmative or negative as the *quality* of a proposition, and the other statuses as the *quantity* of a proposition. Table 1.2 shows the eventual classification.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Affirmative</td>
</tr>
<tr>
<td>UNIVERSAL</td>
<td>Every A is a B</td>
</tr>
<tr>
<td>PARTICULAR</td>
<td>Some A is a B</td>
</tr>
<tr>
<td>INDEFINITE</td>
<td>An A is a B</td>
</tr>
<tr>
<td>SINGULAR</td>
<td>n is a B</td>
</tr>
</tbody>
</table>

5 Many centuries later, Peter of Spain lays out the kinds of non-modal categorical propositions and their ingredients. His terminology was standard:

Of categorical propositions, one is universal, another particular, another indefinite, another singular.

A *universal* proposition is one in which a common term determined by a universal sign is made the subject, like ‘every man runs.’ Or else a universal proposition is one that signifies that something is in every item or none.

A *common* term is one that is naturally suited to be predicated of many, like ‘man’ of Sortes, Plato and each one of the other men.

Universal signs are these: ‘every,’ ‘none,’ ‘nothing,’ ‘any-at-all,’ ‘either,’ ‘neither’ and the like.

A *particular* proposition is one in which a common term determined by a particular sign is made the subject, like ‘some man runs.’

Particular signs are these: ‘some’, ‘a-certain’, ‘the-other’, ‘the-remaining’ and the like.

An *indefinite* proposition is one in which a common term without a sign is made the subject, like ‘man runs.’

A *singular* proposition is one in which a singular term, or a common term joined with a demonstrative pronoun, is made the subject, like ‘Sortes runs’ or ‘that man runs.’

A *singular term* is one that is naturally suited to be predicated of just one item.

Also, of categorical propositions, one is affirmative, another negative.

An *affirmative* categorical proposition is one in which the predicate is affirmed of the subject, like ‘a man runs.’

A *negative* categorical proposition is one in which the predicate is eliminated from the subject, like ‘a man does not run.’ (Peter of Spain *LS* I.8–9 (4))
I call these eight forms “standard” categorical propositions, to distinguish them from the extended forms to be discussed later.

### Applications

Classify each of the following as to quantity and quality.

- A wolf is an animal
- Madonna is a singer
- No actor is a tycoon
- Some actor isn’t a dancer
- George Washington isn’t an actor
- Every actor is a dancer

### 1.2 Logical relations among categorical propositions

Singular propositions: Affirmative and negative singular propositions with the same subject and predicate are contradictories; that is, one of them must be true and the other one false. So given 'Socrates is a philosopher' and 'Socrates isn’t a philosopher' we know as a matter of logic that one of these is true and the other one false, though we may not know which is which. If Socrates actually exists, then 'Socrates is a philosopher' is true if and only if that man, Socrates, is actually a philosopher, and 'Socrates isn’t a philosopher' is true if and only if that man, Socrates, is not a philosopher. In case Socrates does not exist, the affirmative form: 'Socrates is a philosopher' is false, and the negative form: 'Socrates isn’t a philosopher' is true. Since Socrates actually died a long time ago, he does not now exist, and so today 'Socrates is a philosopher' is false and 'Socrates isn’t a philosopher' is true.

Most medieval theorists assume that every well-formed categorical proposition is either true or false (and not both). (The possible exceptions that were discussed are

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6 In *O1* Aristotle uses the word ‘contradictory’ to mean something like what we would call ‘opposite,’ and he investigates under what conditions pairs of ‘contradictories’ must have one member true and the other false. Sometimes they can be true together, as in ‘a man is white’ and ‘a man isn’t white’ (O1 17b30). (See Whitaker 1996 for discussion.) But in the ensuing tradition, ‘contradictory’ is taken to mean having opposite truth values. Ammonius, writing as early as 470 CE, says (81.14–16) “the definition of contradiction . . . is a conflict of an affirmation and a negation which always divide the true and the false so that when one of them is false the other is true, and vice versa.” Peter of Spain (LS 1.14): “The law of contradictories is this, that if one is true, the remaining one is false, and conversely, for they cannot be true or false at the same time with any [subject] matter.”

7 Aristotle *O1* 17b26 (48): “Of contradictory statements about a universal taken universally it is necessary for one or the other to be true or false; similarly if they are about particulars, e.g. ‘Socrates is white’ and ‘Socrates isn’t white.’ (‘particulars’ could as well be translated ‘singulars.’) Categories 13b30–34 (37): “For take ‘Socrates is sick’ and ‘Socrates isn’t sick’: if he exists it is clear that one or the other of them will be true or false, and equally if he does not; for if he does not exist ‘he is sick’ is false but ‘he isn’t sick’ true.”
mostly semantic paradoxes or contingent statements about the future, which are not discussed here.)

In his essay *On Interpretation* Aristotle states that a universal affirmative proposition is the contradictory of the corresponding particular negative proposition, so ‘*Every man is a philosopher*’ is true if and only if ‘*Some man isn’t a philosopher*’ is false, and vice versa. This is supposed to be apparent, given one’s understanding of what those propositions say. He holds the same for the negative case; the universal negative is the contradictory of the corresponding particular affirmative propositions, so that ‘*No man is a philosopher*’ is true if and only if ‘*Some man is a philosopher*’ is false, and vice versa.

Aristotle also holds that corresponding universal propositions are contraries, meaning that they cannot both be true, although they might both be false. For example, ‘*Every man is a philosopher*’ and ‘*No man is a philosopher*’ cannot both be true, though they can both be false. Most logicians today assume that these propositions are not contraries; they are not contraries because they are both true if there are no humans; they are “vacuously” true. Aristotle did not discuss this possibility, but medieval logicians did, and they concluded that Aristotle was right to take them to be contraries. This is because the universal affirmative proposition ‘*Every man is a philosopher*’ is false if there are no men. We will discuss this further later.

### 1.3 The square of opposition

Some commentators on Aristotle found it convenient to use a diagram to keep straight Aristotle’s assumptions about the logic of these propositions. The diagram (see Table 1.3) begins with some of the relationships we have discussed; the universal propositions at the top are contraries, and diagonally opposite propositions are contradictories:

<table>
<thead>
<tr>
<th>Contraries</th>
<th>Contradictories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Every A is B</td>
<td>No A is B</td>
</tr>
<tr>
<td>Some A is B</td>
<td>Some A isn’t B</td>
</tr>
</tbody>
</table>

The doctrines that Aristotle himself laid out entail others. All of these together were typically encapsulated in a square diagram, today called the “square of opposition” (Table 1.4):
Aristotle contributed the top side and the diagonals (and he discussed the bottom at PA 2.15, where he calls the bottom propositions “opposites verbally”). The bottom and side relationships follow from the others. In particular, since the propositions at the top are contraries, their contradictories at the bottom are “subcontraries”: they can both be true but their form prevents them from both being false.8

Universal and particular affirmative propositions are related by subalternation, as are universal and particular negatives: each universal proposition entails the particular directly below it.9 This is easy to see. For the affirmative case, suppose that ‘Every A is B’ is true. Then its contrary, ‘No A is B’ must be false. So the contradictory of ‘No A is B’, namely, ‘Some A is B’ must be true. The reasoning is the same for the negative case: if ‘No A is B’ is true, then its contrary, ‘Every A is B’ must be false, so the contradictory of ‘Every A is B’, which is ‘Some A isn’t B’, is true.

### APPLICATIONS

Say how each of the following pairs of propositions are logically related.

- Some wolf is an animal, Some wolf isn’t an animal
- Madonna is a singer, Madonna isn’t a singer
- No actor is a tycoon, Some actor is a tycoon
- Some actor isn’t a dancer, No actor is a dancer
- George Washington isn’t an actor, George Washington is an actor
- Every actor is a dancer, Some actor isn’t a dancer

---

8 “The law of subcontraries is this, that if one is false, the remaining one is true, and not conversely, for they can both be true at the same time” (Peter of Spain LS 1.14).

9 “The law of subalternates is this, that if the universal is true, the particular is true, and not conversely, for the universal can be false with its particular being true. And if the particular is false, its universal is false, and not conversely” (Peter of Spain LS 1.14).
1.4 Issues concerning empty terms

There are two issues concerning the relations encoded in the square having to do with the truth values of propositions with empty subject terms.

1.4.1 Universal affirmatives

The first issue concerns universal affirmative propositions when the subject term is empty; for example, ‘Every donkey is a mammal’ when there are no donkeys. Suppose that the term ‘A’ is empty.\(^\text{10}\) Then ‘Some A is B’ is false.\(^\text{11}\) According to the principles embodied in the square, ‘No A is B’ is its contradictory, and so it must be true. So, by the law of contraries, the universal proposition ‘Every A is B’ must be false. This goes against the modern custom whereby a universal affirmative proposition with an empty subject term is considered to be trivially true. This is because the canonical translation of ‘Every A is B’ into symbolic logic is ‘∀x(Ax → Bx),’ which is true when ‘A’ is empty.

Modern students often balk at the proposal that universal affirmatives are true when their subject terms are empty. In response they may be told:

This is a convention which is useful in logic—it makes for theoretical simplicity. Ordinary usage is unclear regarding such propositions with empty subjects. If you think that universal affirmatives are false when their subjects are empty, then you may simply represent them by adding a condition: symbolize them as ‘∃xAx & ∀x(Ax → Bx).’

It is apparent that one can also adopt the opposite convention, that universal affirmatives are false when their subject term is empty. This is a convention that is convenient for doing logic—it makes for theoretical simplicity (we return to this shortly). If you want to represent ordinary usage in the contemporary way, as ‘∀x(Ax → Bx),’ then just write ‘Every A is B, or no A is A.’\(^\text{12}\)

1.4.2 Particular negatives

The other issue with the traditional square of opposition concerns particular negatives. Suppose that the term ‘A’ is empty. Then ‘Some A is B’ is false. So according to the principles embodied in the square, ‘No A is B’ is its contradictory, and is thus true. So, by the principle of subalternation, ‘Some A isn’t B’ is also true. But to modern ears, ‘Some A isn’t B’ should be false if ‘A’ is empty. After all, ‘some A,’ has scope over the rest of the proposition. What is going on?

This result is built into the diagram in other ways as well. Again, suppose that ‘A’ is empty, so that ‘Some A is B’ is false. Then its subcontrary, ‘Some A isn’t B’ must be true.

\(^{10}\) By an empty term I mean one that has no individuals falling under it. Aristotle uses ‘goat-stag’ for an example (Posterior Analytics 2.7).

\(^{11}\) This presupposes an “extensional” interpretation of ‘Some A is B’; on this understanding the proposition has exactly the truth conditions assumed in modern logic: ‘∃x(Ax&Bx).’ For an interesting and plausible alternative see Malink 2013.

\(^{12}\) The issue becomes more complicated when propositions become more complex. See section 9.3 for discussion.
Or suppose that ‘Some A is B’ is false; then its superalternate ‘Every A is B’ is also false; so the contradictory of that, ‘Some A isn’t B’, is again true.

Some authors did not notice this result, but many did. Most who noticed held that this is the right result: if ‘A’ is empty, then ‘Some A isn’t B’ is indeed true. This may not accord well with ordinary speakers of Latin, but logicians insisted that this was the right way to read the proposition. This is part of their regimentation of the language that will be discussed later. It may be defended in the way that any regimentation is defended, by claiming that it is useful for logical purposes, even if it does not conform well to ordinary usage.

Of course, this proposal will not work unless other parts of logic are formulated with this in mind. For example, we do not want to include the validity of ‘Some A isn’t B . ∴ Some A is A.’ This inference, of course, is not considered valid.

I said that these proposals make for theoretical simplicity, but I didn’t say how. It is this:

<table>
<thead>
<tr>
<th><strong>Affirmative categorical propositions</strong></th>
<th><strong>Negative categorical propositions</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>are false when any of their main terms are empty.</td>
<td>are true when any of their main terms are empty.</td>
</tr>
</tbody>
</table>

These principles hold for singular propositions and for the forms of categorical propositions we have discussed; they will hold without exception, even when categorical propositions are expanded far beyond the forms that Aristotle discussed.

### APPLICATIONS

Classify each of the following as to truth or falsehood. (Assume that there are now no dodos, that Elvis does not exist, and that Madonna exists.)

<table>
<thead>
<tr>
<th>Some dodo is an animal</th>
<th>Some dodo isn’t an animal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Madonna is a singer</td>
<td>Elvis isn’t a singer</td>
</tr>
<tr>
<td>No dodo is an animal</td>
<td>Every dodo is a dodo</td>
</tr>
</tbody>
</table>

---

13 Abelard (D, p. 170) thought that this is wrong; he held that the particular negative form should be read ‘Not every A is B’ instead of ‘Some A isn’t B’. He blamed the latter “misreading” on Boethius, who wrote the latter form instead of the former, which Aristotle had used. But Aristotle (PA 27a36) uses both forms interchangeably.

14 I think that this interpretation of Aristotle's logic is consistent with his writings. Other writers disagree with this. For example, Robin Smith in the Stanford Encyclopedia of Philosophy on Aristotle’s Logic says: “Aristotle in effect supposes that all terms in syllogisms are non-empty.” (Smith does not give reasons in that work.) This view has a substantial following; it was reached in Kneale and Kneale’s classic 1962: “In order to justify Aristotle’s doctrine as a whole it is necessary, then, to suppose that he assumed application for all the general terms with which he dealt” (60). (I am not certain that I follow their reasoning. I think that it relies on only considering the two options: that universals never have existential import, or that universals, both affirmative and negative, always have existential import.)
1.5 Conversions

The square of opposition deals with logical relations among propositions which have the same subject and same predicate. There are also other relations. If a proposition is generated from another by interchanging the original subject and predicate, those propositions are candidates for a logical relation of "conversion":\(^\text{15}\)

**Simple conversion**: A proposition is said to *convert simply* if it entails the result of interchanging its subject and predicate terms. Universal Negative and Particular Affirmative propositions convert simply, resulting in equivalent propositions:

- ‘No A is a B’ converts simply to ‘No B is an A’
- ‘Some A is a B’ converts simply to ‘Some B is an A’

**Conversion per Accidents** (Accidental conversion): Whereas simple conversion produces a proposition equivalent to the original, conversion *per accidens* is not symmetric. A universal proposition may be converted *per accidens*: you interchange its subject and predicate terms and change the universal sign to a particular sign (adding a negation, if necessary, to preserve quality). This inference is not reversible.

- ‘Every A is a B’ converts *per accidens* to ‘Some B is an A’
- ‘No A is a B’ converts *per accidens* to ‘Some B isn’t an A’\(^\text{16}\)

Since the universal changes to a particular, this form of conversion is sometimes called “conversion by limitation.” Aristotle himself discusses *per accidens* conversion of the universal affirmative form; later writers typically include both forms. (Cf. Roger Bacon, ASL, para 279.) We discuss proofs of both forms in the next chapter.\(^\text{17}\)

### Applications

Say which of the following are valid conversions, which are invalid conversions, and which are not conversions at all.

- Some dodo is an animal \(\therefore\) Some animal is a dodo
- No dodo is an animal \(\therefore\) No animal is a dodo
- Some dodo isn’t an animal \(\therefore\) Some animal isn’t a dodo
- Every dodo is an animal \(\therefore\) Every animal is a dodo
- Every dodo is an animal \(\therefore\) Some animal is a dodo
- Every dodo is an animal \(\therefore\) Some dodo is an animal

\(^{15}\) The conversion laws were proved by Aristotle in Prior Analytics I.2 (2–3). They appear (usually without proof) in every major logic text in the Aristotelian tradition, except that conversion per accidens is sometimes dropped from 20th-century texts.

\(^{16}\) Cf. Peter of Spain, LS I.15, (8). Many authors (including Aristotle) did not mention converting the Universal Negative in this way. This conversion is a consequence of other logical relations assumed in the square. Buridan (SD I.6.3) argues: “We should note that in this type of conversion even a universal negative is validly converted into a particular negative: ‘No B is A; therefore, some A isn’t B.’ For by simple conversion the following is valid: ‘No B is A; therefore no A is B, whence further ‘No A is B; therefore, some A isn’t B’ for a universal implies its subaltern particular; therefore ‘No B is A; therefore, some A isn’t B’ is valid, since whatever follows from the consequent follows from the antecedent.”

\(^{17}\) An additional mode of conversion—conversion by contraposition—is discussed in section 3.5.
1.6 Syllogisms

Syllogisms form the heart of what is today commonly called Aristotelian logic. A syllogism is a special form of argument containing two premises and a conclusion, each of which is a non-singular standard form categorical proposition. There are three distinct terms; one of them, the “middle” term, occurs once in each premise. Each of the other terms occurs once in a premise and once in the conclusion. An example of a syllogism is any argument having the following pattern:

\[
\begin{align*}
\text{Every } M & \text{ is } P \\
\text{Some } M & \text{ is } S \\
\therefore \text{ Some } S & \text{ is } P
\end{align*}
\]

The first premise is called the major premise and the second is called the minor. These individual argument patterns are called “moods,” and the moods in turn are classified into three figures. The first figure includes moods in which the middle term occurs as subject in the first premise and predicate in the second; in the second figure the middle term is predicate in both; and in the third figure it is the subject in both. Aristotle discusses some of the first figure moods as well as the second and third figure moods in chapters 4–6 of *Prior Analytics I*, and he discusses some additional first figure moods in chapter 7, for a total of 19 good moods.

There are five additional valid patterns that neither he nor many medieval authors mention; these are forms which conclude with a particular proposition when the super-alternate universal proposition is provable from the same premises. An example is:

\[
\begin{align*}
\text{Every } A & \text{ is } B \\
\text{Every } C & \text{ is } A \\
\therefore \text{ Some } C & \text{ is } B
\end{align*}
\]

The moods are divided into direct and indirect, where a direct mood is one in which the first premise contains the predicate of the conclusion. In the second and the third figures all of the moods that Aristotle discusses are direct. Interchanging the premises in these figures produces indirect moods that are in the same figure. Since interchanging the premises of a mood cannot affect validity, the results of swapping premises are not listed as additional moods (though in the first figure the direct and indirect moods are separately listed). Sometimes only the direct moods are listed in figure 1, and the

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18 Aristotle also discusses cases in which two terms are identified, so there are four occurrences of one term and two of the other.
19 The term ‘mood’ might better be translated ‘mode,’ but the former term is standard in the tradition.
20 Aristotle has only 19 moods because he is examining which combinations of premises can yield a valid syllogism, and there are 19 of these. Five of those yield more than one conclusion, and thus there are 24 all told if the pattern includes the form of the conclusion. The tradition typically discusses only the forms with the stronger conclusion. E.g. Arnauld and Nicole 1662, 142: “people have been satisfied with classifying syllogisms only in terms of the nobler conclusion, which is the general. Accordingly they have not counted as a separate type of syllogism the one in which only a particular conclusion is drawn when a general conclusion is warranted.”